

Use the following matrices for the problems 1 - 13. **Show ALL WORK by HAND** and check with the calculator.

$$A = \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -2 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix} \quad E = \begin{bmatrix} 5 & x & 3 \\ 0 & 7 & y+1 \end{bmatrix}$$

1. What are the dimensions of Matrix D?

2. $[D]^T = \begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix}^T = \begin{bmatrix} 2 & -4 & 3 \\ 0 & 3 & -2 \end{bmatrix}$

3. $[C] + [E] =$

4. $[D] + [C] =$

NOT POSSIBLE;
DIMENSION
MISMATCH

5. $2[B] - 3[A] =$

INNER
DIMENSIONS
MATCH

6. $[C] \cdot [D] =$

7. $[B] \cdot [D] =$

NOT POSSIBLE;
DIMENSION
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8. $[B] \cdot [E] =$

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9. $C_{22} + D_{31} - B_{21} =$

Row Column Row Column Row Column

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & -2 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix} - \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix}$$

$$-2 + 3 - -7 = 8$$

8

10. $[A]^2 =$

$$\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ -14 & 11 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} -3 \cdot -3 + 2 \cdot -7 & -3 \cdot 2 + 2 \cdot 5 \\ -7 \cdot -3 + 5 \cdot -7 & -7 \cdot 2 + 5 \cdot 5 \end{bmatrix} = \begin{bmatrix} 9 - 14 & -6 + 10 \\ 21 - 35 & -14 + 25 \end{bmatrix} = \begin{bmatrix} -5 & 4 \\ -14 & 11 \end{bmatrix}$$

11. $[D] \cdot [D]^T =$

$$\begin{bmatrix} 2 & -4 & 3 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -8 & 6 \\ -8 & 25 & -18 \\ 6 & -18 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 2 \cdot 2 + 0 \cdot 0 & 2 \cdot -4 + 0 \cdot 3 & 2 \cdot 3 + 0 \cdot -2 \\ -4 \cdot 2 + 3 \cdot 0 & -4 \cdot -4 + 3 \cdot 3 & -4 \cdot 3 + 3 \cdot -2 \\ 3 \cdot 2 + -2 \cdot 0 & 3 \cdot -4 + -2 \cdot 3 & 3 \cdot 3 + -2 \cdot -2 \end{bmatrix} = \begin{bmatrix} 4 + 0 & -8 + 0 & 6 + 0 \\ -8 + 0 & 16 + 9 & -12 + -6 \\ 6 + 0 & -12 + -6 & 9 + 4 \end{bmatrix} = \begin{bmatrix} 4 & -8 & 6 \\ -8 & 25 & -18 \\ 6 & -18 & 13 \end{bmatrix}$$

12. Create a 2x2, 3x3, and 5x5 IDENTITY matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

13. Are Matrices A and B inverses? Demonstrate how you can tell.

IF THEIR PRODUCT RESULTS IN AN IDENTITY MATRIX THEN THEY ARE INVERSES OF EACH OTHER.

$$[A] \cdot [B] = \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -3 \cdot -5 + 2 \cdot -7 & -3 \cdot 2 + 2 \cdot 3 \\ -7 \cdot -5 + 5 \cdot -7 & -7 \cdot 2 + 5 \cdot 3 \end{bmatrix} = \begin{bmatrix} 15 - 14 & -6 + 6 \\ 35 - 35 & -14 + 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[B] \cdot [A] = \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} -5 \cdot -3 + 2 \cdot -7 & -5 \cdot 2 + 2 \cdot 5 \\ -7 \cdot -3 + 3 \cdot -7 & -7 \cdot 2 + 3 \cdot 5 \end{bmatrix} = \begin{bmatrix} 15 - 14 & -10 + 10 \\ 21 - 21 & -14 + 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Determine possible values for a and b that would make the following matrix statement true.

$$14. \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ b \end{bmatrix}$$

$$\begin{aligned} 2a + 12 &= 2 \\ 2a &= -10 \\ a &= -5 \end{aligned}$$

$$\begin{aligned} 2a + 6 &= b \\ 2(-5) + 6 &= b \\ -10 + 6 &= b \\ -4 &= b \end{aligned}$$

$$a = -5$$

$$b = -4$$

Determine possible values for a, b, and c that would make the following matrix statement true.

$$14. \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & 6 \\ 2 & b \end{bmatrix} = \begin{bmatrix} 6 & 27 \\ 8 & c \end{bmatrix}$$

$$\begin{aligned} 4a + 2 &= 6 \\ 4a &= 4 \\ a &= 1 \end{aligned}$$

$$\begin{aligned} 2a + 6 &= 8 \\ 2(1) + 6 &= 8 \\ 2 + 6 &= 8 \\ 8 &= 8 \end{aligned}$$

$$\begin{aligned} 12 + 3b &= c \\ 12 + 3(1) &= c \\ 12 + 3 &= c \\ 15 &= c \end{aligned}$$

$$\begin{aligned} 24 + b &= 27 \\ -24 & \quad -24 \\ b &= 3 \end{aligned}$$

$$\begin{aligned} 12 + 3b &= c \\ 12 + 3(3) &= c \\ 12 + 9 &= c \\ 21 &= c \end{aligned}$$

$$a = 1$$

$$b = 3$$

$$c = 21$$

15. The determinant of $\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$ by hand and show work.

$$\begin{vmatrix} 4 & 2 \\ 5 & 3 \end{vmatrix} = 4 \cdot 3 - 5 \cdot 2 = 12 - 10 = 2$$

$$2$$

14. Find the determinant of $\begin{bmatrix} 3 & 5 & -1 \\ 0 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix}$ by hand and show work.

$$\begin{vmatrix} 3 & 5 & -1 \\ 0 & 2 & 3 \\ 1 & 4 & 6 \end{vmatrix} = 3(12 - 18) - 5(6 - 3) + 1(12 - 2) = 3(-6) - 5(3) + 1(10) = -18 - 15 + 10 = -23$$

$$17$$

15. Find the inverse of $\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$ by hand and show work.

$$\text{IF } [A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ THEN}$$

$$[A]^{-1} = \frac{1}{\det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 3/2 & -1 \\ -5/2 & 2 \end{bmatrix}$$

16. Find the inverse of $\begin{bmatrix} 6 & -3 \\ 4 & 2 \end{bmatrix}$ by hand and show work.

$$\begin{vmatrix} 6 & -3 \\ 4 & 2 \end{vmatrix} = 6 \cdot 2 - 4 \cdot (-3) = 12 + 12 = 24$$

$$\begin{bmatrix} 6 & -3 \\ 4 & 2 \end{bmatrix}^{-1} = \frac{1}{24} \begin{bmatrix} 2 & 3 \\ -4 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} 1/2 & 1/8 \\ -1/6 & 1/4 \end{bmatrix}$$

17. On one weekend, the Goxfield Theater reported the following ticket sales for three first-run movies, as shown in the matrix at the right. If the ticket prices were \$6 for each adult and \$4 for each child, what were the weekend sales for each movie?

	Adults	Children
Movie 1	1021	523
Movie 2	2547	785
Movie 3	3652	2456

	ADULT	CHILD	
	6	4	
MOVIE 1	1021	523	1021 \cdot 6 + 523 \cdot 4
			6126 + 2092
MOVIE 2	2547	785	2547 \cdot 6 + 785 \cdot 4
			15282 + 3140
MOVIE 3	3652	2456	3652 \cdot 6 + 2456 \cdot 4
			21912 + 9824

$$\begin{aligned} \text{MOVIE 1} &= \$8218 \\ \text{MOVIE 2} &= \$18422 \\ \text{MOVIE 3} &= \$31736 \end{aligned}$$

$$\begin{aligned} M1: & \$8218 \\ M2: & \$18422 \\ M3: & \$31736 \end{aligned}$$